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Slope Reliability

Sam I. Thornton

Final Report

1995

SLOPE RELIABILITY

MBTC FR 1014

Sam I. Thornton

TRANSPORTATION

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A reliability approach, using probability calculations which account for the variability in soil strength, is superior to the factor of safety approach. The method is based on the point estimate method and allows engineers to calculate a probability of failure for the slope. Knowing the probability of failure improves engineering judgement by providing a rational basis for making a safe and economical slope design.

Examples show how soil variability affects slope reliability and how the method is applied. The factor of safety is 1.30 in the first two examples. In the first example, the soil deposits are uniform and the probability of failure is acceptable; in the second example, the soils have more soil strength variation and the probability of failure is higher than recommended.

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Slope Reliability

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SLOPE RELIABILITY

SUMMARY

Analysis methods for slope stability are routinely applied by geotechnical engineers. Slope designs, however, are usually based on a "safety factor" which does not account for soil variability (soil variability is due to actual in-place conditions and not due to sampling procedures and/or testing methods). As a result, the true safety of a slope is unknown.

A reliability approach, using probability calculations which account for the variability in soil strength, is superior to the factor of safety approach. The method is based on the point estimate method and allows engineers to calculate a probability of failure for the slope. Knowing the probability of failure improves engineering judgement by providing a rational basis for making a safe and economical slope design.

Examples show how soil variability affects slope reliability and how the method is applied. The factor of safety is 1.30 in the first two examples. In the first example, the soil deposits are uniform and the probability of failure is acceptable; In the second example, the soils have more soil strength variation and the probability of failure is higher than recommended.

1 INTRODUCTION

Geotechnical engineers routinely calculate a factor of safety (FS) to evaluate the stability of earth slopes. The Simplified Bishop method (Wright, et al, 1973) is a popular basis for computer analysis programs. A minimum FS of 1.3 is commonly considered as the design basis for most slopes. Failure is assumed to occur when the FS is less than 1.0.

Because the FS analysis does not have a way to consider the variability of the soil strength, the true safety of a slope is unknown. A reliability approach, where a probability of failure is calculated, is a better method for slope design because it accounts for variability in soil strengths. Other factors, like an inadequate field investigation, missing a critical geologic detail (Christian, et, al., 1994) or progressive slope failures (Chowdhury, R,. N., Sept. 1994) are not included in the method described in this report.

The probability of slope failure method is based on the "Point Estimate Method" (PEM) which was developed by Rosenblueth (1975 and 1981) and described by Harr (1987). In the PEM method, a distribution of the variable must be found or assumed. If a normal distribution is assumed, the problem is simplified. Details of the PEM method and a discussion of other distributions are contained in a thesis by Garrett (1989) and a paper by McGuffy, Iori, Kyfor and Grivas (1981).

2 APPLICATION OF THE POINT ESTIMATE METHOD TO SLOPE STABILITY

2.1 MEAN AND STANDARD DEVIATION

To apply the PEM, the mean and standard deviation of the soil strength in each layer must be found. Soil strength may be cohesion, C, and/or internal friction, ϕ. Between layers, strength parameters are considered independent. Within a soil layer, however, the cohesion may be correlated to the internal friction.

$$Mean, \; \bar{x} \; = \; \frac{x_1 \; + \; x_2 \; + \; x_3 \; + \; ... \; x_n}{n} \qquad \qquad (Eqn. \; 1)$$

$$Standard \; Deviation, \; \sigma \; = \; \sqrt{\frac{\Sigma \; (x_i \; - \; \bar{x})^2}{(n \; - \; 1)}} \qquad \qquad (Eqn. \; 2)$$

where,   x = the C or φ values in the layer
n = the number of C or φ (tests performed) values in the layer

2.2 CORRELATION COEFFICIENT

For a soil layer with C and φ, the correlation between C and φ, must be found. Correlations vary with the type of strength test. For the consolidated undrained triaxial test, Harr (1987) reports a correlation, r, of about +0.25. A positive correlation means the internal friction increases when the cohesion increases. The undrained triaxial test is the best predictor for quick failures caused by earthquakes or the sudden drawdown of water at a levee or dam. Drained triaxial tests often have negative correlations and are usually the best predictor of field performance. Wolff reported a drained triaxial correlation of -0.47 (Harr, 1987). The correlation coefficient, r, is calculated by the following:

$$Correlation\ Coefficient,\ r = \frac{N\ \Sigma(C \cdot \phi) - \Sigma C\ \Sigma\phi}{\sqrt{[N\ \Sigma C^2 - (\Sigma C)^2\ ] \times [N\ \Sigma\phi^2 - (\Sigma\phi)^2]}} \qquad (Eqn.\ 3)$$

where,   N = the number of strength tests

2.3 HIGH AND LOW STRENGTH VALUES

Variation in C and φ is accounted for by adding or subtracting the standard deviation. For example, a high cohesion, C+, is obtained by adding the standard deviation of the cohesion to the mean. A low cohesion, C-, is the mean less the standard deviation. In turn, φ+ and φ- is the mean internal friction + or - the standard deviation of internal friction.

$$C_+ = \overline{C} + \sigma_C \qquad (Eqn.\ 4a)$$
$$C_- = \overline{C} - \sigma_C \qquad (Eqn.\ 4b)$$

where, $\overline{C}$ = mean value of cohesion

$$\phi_+ = \overline{\phi} + \sigma_\phi \qquad (Eqn.\ 5a)$$
$$\phi_- = \overline{\phi} - \sigma_\phi \qquad (Eqn.\ 5b)$$

where,   φ = mean value of internal friction

2.4 SLOPE SAFETY FACTORS

Safety Factors must be found for all combinations of soil strength. The number of

combinations is 2^n, where n is the number of variables (soil strengths). A slope with two layers, each layer with a C and ϕ, has 2^4 or 16 combinations of soil strength. The set of safety factors reflects the variation of soil strength. The symbol FS++++ is used for a slope containing two soil layers with C+ and ϕ+ used for strength values in both layers. FS-+++ is the symbol for the FS when C- and ϕ+ are used for the first layer and C+ and ϕ+ are used for the second layer.

2.5 WEIGHING FUNCTIONS

Weighing functions must be applied to the FS's. The weighing functions are point estimates, p, of the distribution of the FS's. The symbol p++++ is used for the point applied to FS++++ as described in section 2.4. The sum of the p's is equal to 1.

2.51 Independent Layers

For the case where each soil layer has only a C (a clay) or ϕ (a sand) the soil strengths are not correlated. If normal distribution is assumed, the point estimates are:

$$p = 1/2^n \qquad (Eqn.\ 6)$$

$$where, \qquad n = the\ number\ of\ variables\ (layers\ when\ each\ layer\ has\ only\ a\ C\ or\ \phi).$$

The points for two soil layers with C or ϕ are:

$$p++ = p+- = p-+ = p-- = 1/4$$

For three soil layers with C or ϕ, the points are:

$$p+++ = p++- = p+-+ = p-++ = p+-- = p-+- = p--+ = p--- = 1/8$$

The points for four soil layers are:

$$p++++ = p+++- = p++-+ = p+-++ = p-+++ = p++-- =$$
$$p+--+ = p--++ = p+--- = p-+-- = p--+- = p---+ = p-++- =$$
$$p+-+- = p---- = p-+-+ = 1/16$$

2.52 Correlated Layers

When a slope has a single layer with both C and ϕ (two variables), the points are:

$$p++ = p-- = (1 + r)/4 \qquad (Eqn.\ 7a)$$

$$p+- = p-+ = (1 - r)/4 \qquad \text{(Eqn. 7b)}$$

A slope that has two soil layers, each with C and ϕ (four variables), will have the following points:

$$p++++ = p---- = p++-- = p--++ = (1+r_1+r_2)/16 \qquad \text{(Eqn.8a)}$$

$$p+++- = p---+ = p++-+ = p--+- = (1+r_1-r_2)/16 \qquad \text{(Eqn.8b)}$$

$$p+--- = p-+++ = p+-++ = p--+- = (1-r_1+r_2)/16 \qquad \text{(Eqn.8c)}$$

$$p+-+- = p-+-+ = p+--+ = p-++- = (1-r_1-r_2)/16 \qquad \text{(Eqn.8d)}$$

2.53 Mixed Layers

For the case where there are two layers of soil, one layer contains either C or ϕ and the other contains both C and ϕ, the points are:

$$p+++ = p+-- = p--- = p-++ = (1+r_2)/8 \qquad \text{(Eqn.9a)}$$

$$p++- = p+-+ = p--+ = p-+- = (1-r_2)/8 \qquad \text{(Eqn.9b)}$$

2.6 STANDARD DEVIATION OF THE FS'S

The expected value of the factor of safety, E[FS], and the expected value of the squared FS's, must be found in order to calculate the standard deviation of the FS, σ[FS].

2.61 Two Variables

For a slope with two variables (either two layers with C or ϕ, or one layer with C and ϕ):

$$\begin{aligned} E[FS] = &\; p++(FS++) + p+-(FS+-) \\ &+ p-+(FS-+) + p--(FS--) \end{aligned} \qquad \text{(Eqn.10a)}$$

$$\begin{aligned} E[FS^2] = &\; p++(FS++)^2 + p+-(FS+-)^2 \\ &+ p-+(FS-+)^2 + p--(FS--)^2 \end{aligned} \qquad \text{(Eqn.10b)}$$

$$\sigma[FS] = (E[FS^2] - E[FS]^2)^{.5} \qquad \text{(Eqn.11)}$$

2.62 Three Variables

For a slope with three variables (either three layers with C or ϕ, or two layers; one layer with C or ϕ, and one layer with C and ϕ):

$$E[FS] = p+++(FS+++) + p++-(FS++-)$$
$$+ p+--(FS+--) + p---(FS---)$$
$$+ p--+(FS--+) + p-++(FS--+)$$
$$+ p-+-(FS-+-) + p+-+(FS+-+) \qquad (Eqn.12a)$$

$$E[FS^2] = p+++(FS+++)^2 + p++-(FS++-)^2$$
$$+ p+--(FS+--)^2 + p---(FS---)^2$$
$$+ p--+(FS--+)^2 + p-++(FS-++)^2$$
$$+ p-+-(FS-+-)^2 + p+-+(FS+-+)^2 \qquad (Eqn.12b)$$

$$\sigma[FS] = (E[FS^2] - E[FS]^2)^{.5} \qquad (Eqn.11)$$

2.63 Four or More Variables

For four or more variables, the expected FS, E[FS], is found by multiplying the points, p, by their respective FS's and summing the products (see equations 10a and 12a).

The E[FS²] is found by multiplying the points, p, by their respective squared FS's and summing the products (see equations 10b and 12b).

The standard deviation is found from equation 11.

2.7 PROBABILITY OF FAILURE

For normal distribution, the standardized variable Z is:

$$Z = (FS - E[FS])/\sigma[FS] \qquad (Eqn.\ 13)$$

where, $\qquad$ *FS = the cutoff value to be evaluated (FS = 1)*

E[FS] and σ[FS] are found from section 2.6.

With Z, the probability that the FS will be less than 1 can be found from the normal distribution table in Appendix A.

3 ACCEPTABLE FAILURE PROBABILITIES

In order to evaluate a design, the calculated probability of failure should be compared to an acceptable probability. A table of acceptable failure probabilities was proposed by Santamarina, et. al. (1992). A partial listing of the table is contained in Table 1.

TABLE 1. Slope Stability - Probability of Failure

CONDITIONS	P_f
Unacceptable in most cases	>0.1
Temporary structures with low repair cost	0.1
Low consequences of failure repairs when time permits	0.02
Existing large cut on interstate highway	0.01
Acceptable in most cases except if lives may be lost	0.001
Acceptable for all slopes	0.0001
Unnecessarily low	0.00001

4 EXAMPLES

4.1 CONVERSION FACTORS

SI to English	English to SI
1 m = 3.281 ft	1 ft = 0.3048 m
1 kN/m^2 = 20.885 lb/ft^2	1 lb/ft^2 = 0.04788 kN/m^2
1 kN/m^3 = 6.361 lb/ft^3	1 lb/ft^3 = 0.1572 kN/m^3

4.2 TWO LAYERS WITH EITHER C OR Φ

Two examples using the slope in Figure 1 will show how the method is applied. The unit weight of both soil layers is 20 kN/m^3
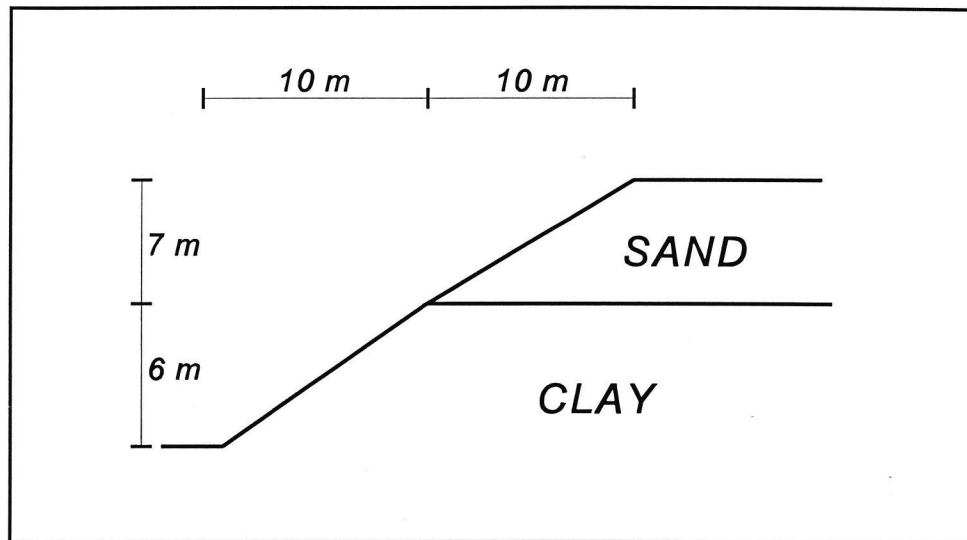
FIGURE 1

4.21 *Example 1: Slope with Uniform Soils*

The internal friction, ϕ, and cohesion, C, from tests for the soil in Figure 1 are:

SAND	CLAY
$\phi°$	C (kN/m²)
33.5	60
36.5	63
35.5	64
34.5	58
35.1	62.5
34.9	

The **mean** (Eqn.1) and **standard deviation** (Eqn.2) are as follows:

SAND	CLAY
mean ϕ = 35°	mean C = 61.5 kN/m²
σ (ϕ) = 1°	σ (C) = 2.45

The **high and low strength values** (Eqn. 4a, 4b, 5a, and 5b) used to determine slope stability factors of safety are:

$\phi+$ = 36°	C+ = 63.95 kN/m²
$\phi-$ = 34°	C- = 59.05 kN/m²

The **strength combinations** for slope stability analysis are:

$$
\begin{array}{lll}
++ \text{ Sand } \phi & = 36° & \text{Clay } C = 63.95 \text{ kN/m}^2 \\
+- \text{ Sand } \phi & = 36° & \text{Clay } C = 59.05 \text{ kN/m}^2 \\
-+ \text{ Sand } \phi & = 34° & \text{Clay } C = 63.95 \text{ kN/m}^2 \\
-- \text{ Sand } \phi & = 34° & \text{Clay } C = 59.05 \text{ kN/m}^2
\end{array}
$$

The resulting **factors of safety** from the computer program PCSTABL5 (Bishop Method) are:

$$
\begin{array}{ll}
++ \text{ FS} & = 1.350 \\
+- \text{ FS} & = 1.248 \\
-+ \text{ FS} & = 1.348 \\
-- \text{ FS} & = 1.246
\end{array}
$$

The **weighing functions** (Eqn. 6) for two soil types with 1 strength parameter per layer is:

$$p++ = p+- = p-+ = p-- = 0.25$$

The **expected FS** (Eqn. 10a) is:

$$
\begin{aligned}
E[FS] &= 0.25(1.350) + 0.25(1.248) + 0.25(1.348) + 0.25(1.246) \\
&= 1.298
\end{aligned}
$$

The **expected FS^2** (Eqn. 10b) is:

$$
\begin{aligned}
E[FS^2] &= 0.25(1.350)^2 + 0.25(1.248)^2 + 0.25(1.348)^2 + 0.25(1.246)^2 \\
&= 1.6874
\end{aligned}
$$

The **standard deviation of the FS's** (Eqn. 11) is:

$$
\begin{aligned}
\sigma[FS] &= [(1.687) - (1.298)^2)]^{.5} \\
&= 0.051
\end{aligned}
$$

The **standardized variable** (Eqn. 13) is:

$$Z = (1-1.298)/0.051 = -5.84$$

For a FS = 1, where failure is assumed to occur, the **probability of failure**, P_f, is (Appendix A):

$$P_f < 0.0000001$$

This probability of failure, according to Table 1, is unnecessarily low.

4.22 Example 2: Slope with Variable Clay

Strength test results for the soil in Figure 1 are as follows:

Sand $\phi°$	Clay $C(kN/m^2)$
36.5	55
34	50
34.5	71
35.5	82
34.5	53
	58

The **mean** (Eqn. 1) and **standard deviation** (Eqn. 2) are as follows:

Sand	Clay
mean ϕ = 35°	mean C = 61.5 kN/m²
$\sigma (\phi)$ = 1°	$\sigma (C)$ = 12.4 kN/m²

The **high and low strength values** (Eqn. 4a, 4b, 5a, and 5b) used to determine slope stability factors of safety are:

$\phi+$ = 36°   C+ = 73.9 kN/m²
$\phi-$ = 34°   C- = 49.1 kN/m²

The **strength combinations** for slope stability analysis are:

++ Sand ϕ = 30°   Clay C = 73.9 kN/m²
+- Sand ϕ = 36°   Clay C = 49.1 kN/m²
-+ Sand ϕ = 34°   Clay C = 73.9 kN/m²
-- Sand ϕ = 34°   Clay C = 49.1 kN/m²

The **factors of safety** from the computer program PCSTABL5 (Bishop Method) are:

++ FS = 1.556
+- FS = 1.040
-+ FS = 1.554
-- FS   = 1.039

The **expected FS** (Eqn. 10a) is:

$$E[FS] = 1.297$$

The **standard deviation of the FS's** (Eqn. 11) is:

$$\sigma[FS] = 0.2578$$

The **standardized variable** (Eqn. 13) and the **probability of failure** (Appendix A) are:

$$Z = (1-1.297)/0.2578 = -1.1637$$
$$P_f = 0.125$$

This probability of failure, according to Table 1 is too high, even for temporary structures with low repair costs.

4.23 Example Comparison

The probability that the slopes in the two examples would fail is greatly different; less than 0.00001% for the first example vs. 12.5% for the second example. This difference is surprising because the geometry unit weight, and average strength of the soil layers within the slopes are the same. The reason for the difference in probability of failure is the variability in cohesion of the clay layer. In the uniform clay layer (section 4.11) the standard deviation of the cohesion is 2.45 kN/m^2 or 4% of the average cohesion. The variable clay layer (section 4.12) has a standard deviation of 12.4 kN/m^2 or 20% of the average cohesion.

4.3 EXAMPLE 3: THREE LAYERS WITH EITHER C OR Φ

The figure below is a slope on Interstate 40 near Morrilton, Arkansas. The slope has been divided into 3 layers.
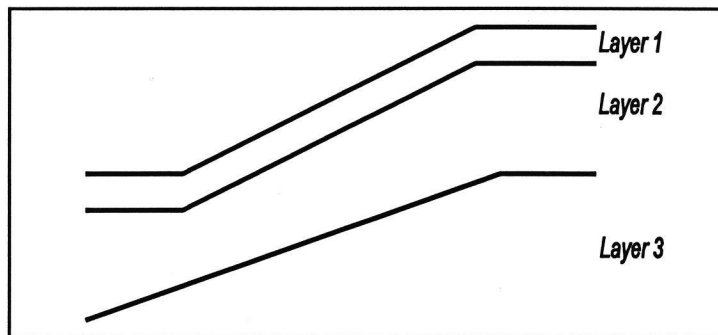

FIGURE 2

In this example, the only strength parameter in each layer is cohesion. From the strength tests, the **mean** and **standard deviation** of each layer obtained from Eqn. 1 and 2 are:

LAYER NO.	MEAN STRENGTH	STANDARD DEVIATION
1	180 lb/ft^2	16 lb/ft^2
2	410 lb/ft^2	54 lb/ft^2
3	600 lb/ft^2	138 lb/ft^2

The **high and low values** (Eqn. 4a and 4b) for cohesion in lb/ft^2 are:

$$\underline{\text{Layer 1}} \qquad \underline{\text{Layer 2}} \qquad \underline{\text{Layer 3}}$$
$$C_1+ = 196 \qquad C_2+ = 464 \qquad C_3+ = 738$$
$$C_1- = 164 \qquad C_2- = 356 \qquad C_3- = 462$$

The next step is putting together the strength combinations. In this case, since there are 3 strength parameters, the are 2^3, or 8 strength combinations.

The **strength combinations** and **factors of safety** from the computer program PCSTABL5 (Bishop Method) for each combination are as follows:

COMBINATION	C_1	C_2	C_3	FS
+++	196	464	738	1.466
++-	196	464	462	1.293
+-+	196	356	738	1.145
-++	164	464	738	1.452
+--	196	356	462	1.131
-+-	164	464	462	1.285
--+	164	356	738	1.145
---	164	356	462	1.131

The next step is the calculation of the **expected FS** (Eqn. 12a), **expected value of the squared FS's** (Eqn. 12b), and **standard deviation of the FS's** (Eqn. 11):

$$E[FS] = 1.256$$
$$E[FS^2] = 1.595$$
$$\sigma[FS] = 0.1326$$

Then the **standardized variable** (Eqn. 13) is found for a FS = 1.

$$Z = (1-1.256)/0.1326 = 1.93$$

By using this Z and the probability chart in Appendix A, the **probability of failure** for this slope is 2.68%.

4.4 EXAMPLE 4: FOUR LAYERS WITH C OR Φ

The example for four layers of soil is taken from the thesis at the University of Arkansas by Steven Garrett (1989). Figure 3 contains the geometry of the slope.
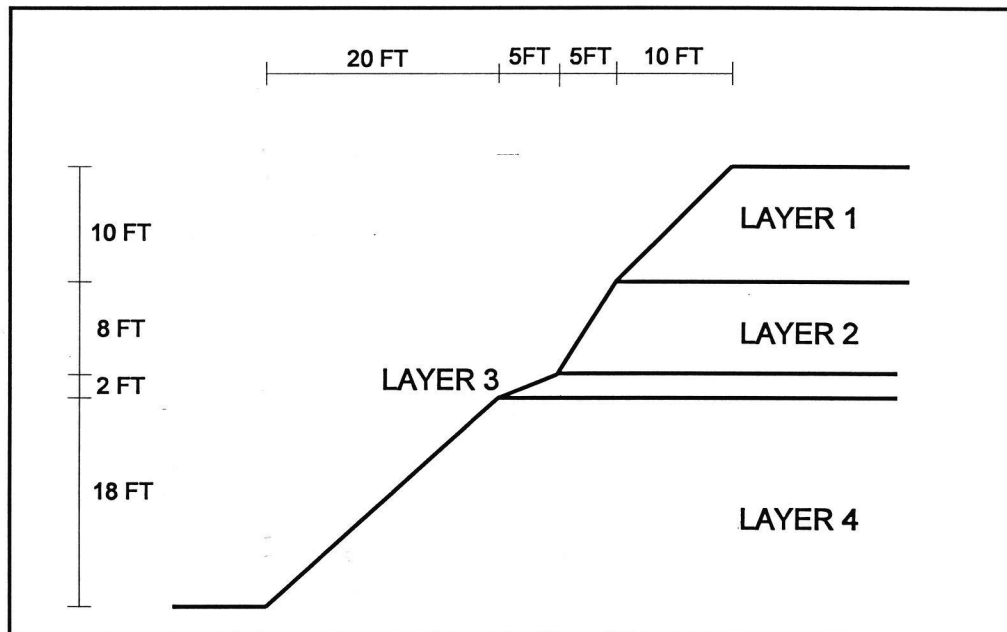

FIGURE 3

In this example, the first and third layers are clay and the second and fourth layers are sand. From the strength tests, the **mean** (Eqn.1) and **standard deviation** (Eqn. 2) of strengths are:

LAYER	MEAN STRENGTH	STANDARD DEVIATION
1	3500 lb/ft^2	200 lb/ft^2
2	27°	5°
3	2000 lb/ft^2	300 lb/ft^2
4	32°	2.5°

The **high and low strength values** (Eqn. 4a, 4b, 5a, and 5b) are:

LAYER	HIGH STRENGTH	LOW STRENGTH
1	3700 lb/ft^2	3300 lb/ft^2
2	32°	22°
3	2300 lb/ft^2	1700 lb/ft^2
4	34.5°	29.5°

The **strength combinations** and **factors of safety** from the computer program PCSTABL5 (Bishop Method) for each combination are:

COMBINATION	C_1 lb/ft^2	ϕ_2 °	C_3 lb/ft^2	ϕ_4 °	FS
++++	3700	32	2300	34.5	1.4024
+++-	3700	32	2300	29.5	1.1966
++--	3700	32	1700	29.5	1.1428
+---	3700	22	1700	29.5	1.1239
----	3300	22	1700	29.5	1.1235
-+--	3300	32	1700	29.5	1.1424
-++-	3300	32	2300	29.5	1.1966
-+++	3300	32	2300	34.5	1.4021
--+-	3300	22	2300	29.5	1.1798
--++	3300	22	2300	34.5	1.3786
---+	3300	22	1700	34.5	1.1798
-+-+	3300	32	1700	34.5	1.3352
+-+-	3700	22	2300	29.5	1.1798
+--+	3700	22	1700	34.5	1.3130
+-++	3700	22	2300	34.5	1.3790
++-+	3700	32	1700	34.5	1.3356

The **expected FS, expected value of the squared FS's,** and **standard deviation of the FS's** are found per article 2.63:

$$E[FS] = 1/16 \, [1.4024 + 1.1966 + 1.1428 + 1.1239 + 1.1235 + 1.1424 +$$
$$1.1966 + 1.4021 + 1.1798 + 1.3786 + 1.3126 + 1.3352 +$$
$$1.1798 + 1.3130 + 1.3790 + 1.3356]$$
$$= 1.2590$$

$$E[FS^2] = 1/16 \, [1.4024^2 + 1.1966^2 + 1.1428^2 + 1.1239^2 + 1.1235_2 + 1.1424^2 +$$
$$1.1966^2 + 1.4021^2 + 1.1798^2 + 1.3786^2 + 1.3126^2 + 1.3352^2 +$$
$$1.1797^2 + 1.3130^2 + 1.3790^2 + 1.3356^2]$$
$$= 1.5958$$

$$\sigma[FS] = (1.5958 - 1.2590^2)^{.5}$$
$$= 0.1035$$

Then the **standardized variable** is found for a FS = 1.

$$Z = (1.0 - 1.2590)/0.1305$$
$$= 2.50$$

Using the probability chart in Appendix A, the **probability of failure** is 0.62%.

4.5 EXAMPLE 5: ONE SOIL WITH TWO VARIABLES

This example is taken from a paper by Verduin and Lovell (1988). The embankment is 40 feet high and is built on a slope of two horizontal to one vertical (Figure 4). The soil has a unit weight of 140 lb/ft³.
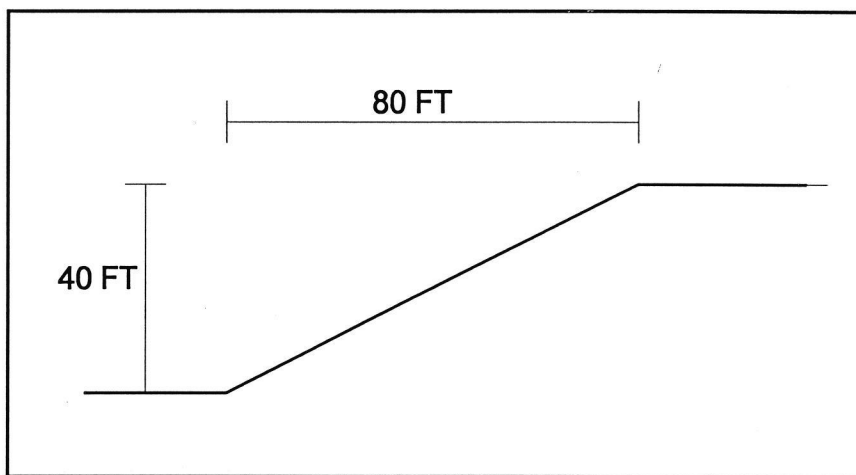

FIGURE 4

The **mean** and **standard deviation** of the soil strength are:

$$\text{mean C} = 200 \text{ lb/ft}^2 \qquad \sigma(C) = 80 \text{ lb/ft}^2$$
$$\text{mean } \phi = 25° \qquad \sigma(\phi) = 2.5°$$

The **correlation coefficient** (Eqn. 3) as determined from laboratory tests is +0.25.

The **high and low strength values** (Eqn 4a, 4b, 5a, and 5b) used to determine slope stability factors of safety are:

$$\phi+ = 25 + 2.5 = 27.5° \qquad C+ = 200 + 80 = 280 \text{ lb/ft}^2$$
$$\phi- = 25 - 2.5 = 22.5° \qquad C- = 200 - 80 = 120 \text{ lb/ft}^2$$

The slope **factors of safety** from the computer program PCSTABL5 (Bishop Method) are:

$$FS++ = 1.685$$
$$FS+- = 1.454$$
$$FS-+ = 1.373$$
$$FS-- = 1.140$$

The **weighing functions** (Eqn. 7a and 7b) are:

$$p++ = p-- = 0.25(1+0.25) = 0.3125$$
$$p+- = p-+ = 0.25(1-0.25) = 0.1875$$

The **expected FS** (Eqn. 10a), **expected value of the squared FS's** (Eqn.10b), and **standard deviation of the FS's** (Eqn. 11) are:

$$E[FS] = 0.3125(1.685) + 0.1875(1.454) +$$
$$0.1875(1.373) + 0.3125(1.140)$$
$$= 1.413$$

$$E[FS^2] = 0.3125(1.685)^2 + 0.1875(1.454)^2 +$$
$$0.1875(1.373)^2 + 0.3125(1.140)^2$$
$$= 2.043$$

$$\sigma[FS] = (2.043 - (1.413)^2)^{.5}$$
$$= 0.216$$

The **standardized variable** (Eqn. 13) and **probability of failure** (Appendix A) are:

$$Z = (1.0 - 1.413)/0.216 = -1.91$$
$$P_f = 2.8\%$$

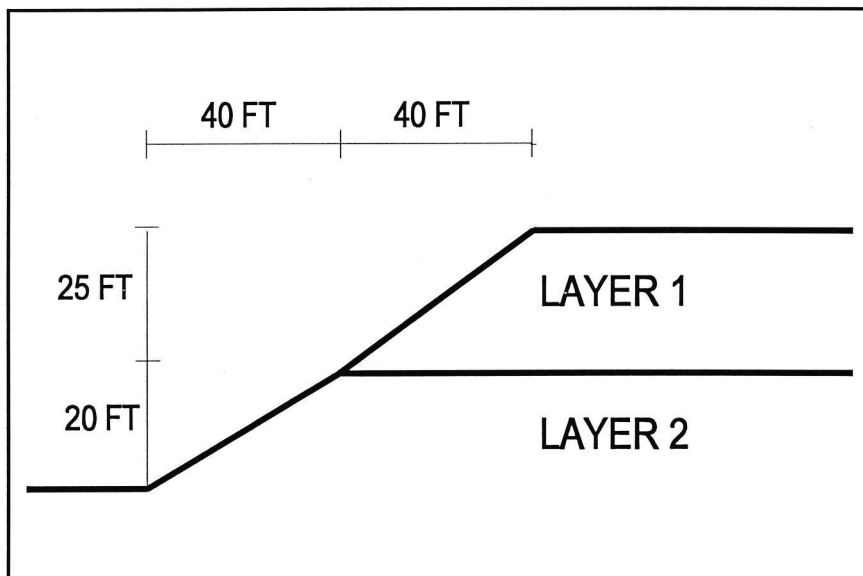## 4.6 TWO SOIL LAYERS WITH TWO VARIABLES EACH

FIGURE 5

Unit Weight Layer 1 = 110 lb/ft^3
Unit Weight Layer 2 = 120 lb/ft^3

The **mean** (Eqn. 1) and **standard deviation** (Eqn. 2) of the soil strength are:

First Layer		Second Layer	
C (lb/ft^2)	$\phi°$	C (lb/ft^2)	$\phi°$
200	31	150	27
180	33	110	30
210	28	240	24
230	27	220	25
160	34	120	32

Layer 1   mean C = 196 lb/ft^2 $\sigma(C)$ = 27 lb/ft^2
           mean ϕ = 30.6° $\sigma(\phi)$ = 3.05°

Layer 2   mean C = 168 lb/ft^2 $\sigma(C)$ = 58.9 lb/ft^2
           mean ϕ = 27.6° $\sigma(\phi)$ = 3.36°

The **correlation coefficients** (Eqn. 3) are -0.964 for layer 1 and -0.927 for layer 2.

The **high and low strength values** (Eqn. 4a, 4b, 5a, and 5b) are:

$$C_1+ = 223 \text{ lb/ft}^2 \qquad \phi_1+ = 33.65°$$
$$C_1- = 169 \text{ lb/ft}^2 \qquad \phi_1- = 27.55°$$
$$C_2+ = 226.9 \text{ lb/ft}^2 \qquad \phi_2+ = 30.96°$$
$$C_2- = 109.1 \text{ lb/ft}^2 \qquad \phi_2- = 24.24°$$

The slope **factors of safety** from the computer program PCSTABL5 (Bishop Method) are:

$$FS++++ = 1.6235$$
$$FS+++- = 1.3798$$
$$FS++-- = 1.2123$$
$$FS+--- = 1.1714$$
$$FS---- = 1.1413$$
$$FS---+ = 1.3573$$
$$FS--++ = 1.5226$$
$$FS-+++ = 1.5897$$
$$FS-+-+ = 1.4579$$
$$FS+-+- = 1.3295$$
$$FS+--+ = 1.3977$$
$$FS-++- = 1.3527$$
$$FS++-+ = 1.4560$$
$$FS--+- = 1.3014$$
$$FS+-++ = 1.5595$$
$$FS-+-- = 1.1873$$

The **weighing functions** (Eqn. 8a, 8b, 8c, and 8d) are:

$$p++++ = p---- = p++-- = p--++ = (1-0.964-0.927)/16 = -0.05569$$
$$p+++- = p---+ = p++-+ = p--+- = (1-0.964+0.927)/16 = 0.0602$$
$$p+--- = p-+++ = p+-++ = p-+-- = (1+0.964-0.927)/16 = 0.0648$$
$$p+-+- = p-+-+ = p+--+ = p-++- = (1+0.964+0.927)/16 = 0.1807$$

The **expected FS, expected value of the squared FS's,** and **standard deviation of the FS's** are:

$$
\begin{aligned}
E[FS] = \ & -0.05569(1.6235) + 0.0602(1.333798) + -0.05569(1.2123) \\
& + 0.0648(1.1714) + -0.05569(1.1413) + 0.0602(1.3573) \\
& + -0.05569(1.5226) + 0.0648(1.5897) + 0.1807(1.4579) \\
& + 0.1807(1.3527) + 0.1807(1.3977) + 0.0602(1.4560) \\
& + 0.0602(1.3014) + 0.1807(1.3295) + 0.0648(1.5595) \\
& + 0.0648(1.1873) \\
= \ & 1.3821
\end{aligned}
$$

$$E[FS^2] = -0.05569(1.6235_2) + 0.0602(1.3798_2) + -0.05569(1.2123_2)$$
$$+ 0.0648(1.1714_2) + -0.05569(1.1413_2) + 0.0602(1.3573_2)$$
$$+ -0.05569(1.5226_2) + 0.0648(1.5897_2) + 0.1807(1.4578_2)$$
$$+ 0.1807(1.3527_2) + 0.1807(1.3977_2) + 0.0602(1.4560_2)$$
$$+ 0.0602(1.3014_2) + 0.1807(1.3295_2) + 0.0648(1.5595_2)$$
$$+ 0.0648(1.1873_2)$$
$$= 1.9136$$

$$\sigma[FS] = (1.9136 - 1.3821^2)^{.5} = 0.05798$$

The **standardized variable** (Eqn. 13) is:

$$Z = (1.0 - 1.382)/0.058$$
$$= -6.59$$

The **probability of failure** (Appendix A) is less than .003%.

5 ARKANSAS SITES

Two sites were selected by the Arkansas Highway and Transportation Department for possible application of the method to existing slopes. The sites were at Highway 67 at State Highway 14 near Newport and I-40 at mile 103.79 near Morrilton.

5.1 NEWPORT

The embankment at Newport was constructed on a 3 horizontal to 1 vertical slope rising 28 feet. Two soil layers were contained in the slope. The first soil layer was a fill 28 feet in height. The second soil layer was the subsoil which was at ground level. Soil characteristics for the layers are:

	SOIL 1 $\phi°$	SOIL 2 $\phi°$
Mean	37.9	30.5
Std. Deviation	2.84	1.26
$\phi+$	40.74	31.76
$\phi-$	35.06	29.24

The resulting **factors of safety** from the computer program PCSTABL5 are:

$$FS++ = 2.763$$
$$FS+- = 2.658$$

$$FS-+ = 2.384$$
$$FS-- = 2.315$$

The **functions** (Eqn. 6) for two soil types with 1 strength parameter per layer is:

$$p++ = p+- = p-+ = p-- = 0.25$$

The **expected FS** (Eqn.10a) is:

$$E[FS] = 0.25(2.763) + 0.25(2.658) + 0.25(2.384) + 0.25(2.315)$$
$$= 2.53$$

The **expected FS²** (Eqn. 10b) is:

$$E[FS^2] = 0.25(2.763)^2 + 0.25(2.658)^2 + 0.25(2.384)^2 + 0.25(2.315)^2$$
$$= 6.43$$

The **standard deviation of the FS's** (Eqn. 11) is:

$$\sigma [FS] = [(6.34) - (2.53)^2)]^{.5}$$
$$= 0.1859$$

The **standardized variable** (Eqn. 13) is:

$$Z = (1-2.53)/0.1859 = -8.23$$

For a FS = 1, where failure is assumed to occur, the **probability of failure**, P_f, is (Appendix A):

$$P_f < 0.0002$$

This probability of failure, according to Table 1, is acceptable for this slope.

5.2 MORRILTON

The Morrilton site, based on the strength data supplied by the Arkansas Highway and Transportation Department, had an expected FS of 3.58. Because the FS is so high, the slope was not analyzed for a probability of failure.
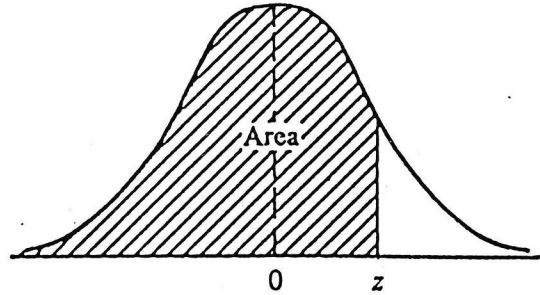
6 CONCLUSION

The reliability approach to slope stability is superior to the safety factor approach because it accounts for variability in soil strength.

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APPENDIX A

NORMAL DISTRIBUTION CURVE AREAS

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
−3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
−3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
−3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
−3.1	0.0010	0.0009	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0007	0.0007
−3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
−2.9	0.0019	0.0018	0.0017	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
−2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
−2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
−2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
−2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
−2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
−2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
−2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
−2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
−2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
−1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
−1.8	0.0359	0.0352	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
−1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
−1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
−1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
−1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0722	0.0708	0.0694	0.0681
−1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
−1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003.	0.0985
−1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
−1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
−0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
−0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
−0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
−0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
−0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
−0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
−0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
−0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
−0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
−0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

Slope Stability Manual

FR-1014

SLOPE FAILURE DESIGN MANUAL

MBTC FR 1014

Sam I. Thornton

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A reliability approach, using probability calculations which account for the variability in soil strength, is superior to the factor of safety approach. The method is based on the point estimate method and allows engineers to calculate a probability of failure for the slope. Knowing the probability of failure improves engineering judgement by providing a rational basis for making a safe and economical slope design.

Examples show how soil variability affects slope reliability and how the method is applied. The factor of safety is 1.30 in the first two examples. In the first example, the soil deposits are uniform and the probability of failure is acceptable; In the second example, the soils have more soil strength variation and the probability of failure is higher than recommended.

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INTERIM REPORT on SLOPE RELIABILITY

Slope Failure Design Manual

Submitted

to

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University of Arkansas, Fayetteville

and

Arkansas Highway and Transportation Department
Little Rock, Arkansas

Prepared

by

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University of Arkansas

March 1995

SLOPE FAILURE DESIGN MANUAL

SUMMARY

Analysis methods for slope stability are routinely applied by geotechnical engineers. Slope designs, however, are usually based on a "safety factor" which does not account for soil variability (soil variability is due to actual inplace conditions and not due to sampling procedures and/or testing methods). As a result, the true safety of a slope is unknown.

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Examples show how soil variability affects slope reliability and how the method is applied. The factor of safety is 1.30 in the first two examples. In the first example, the soil deposits are uniform and the probability of failure is acceptable; In the second example, the soils have more soil strength variation and the probability of failure is higher than recommended.

1 INTRODUCTION

Geotechnical engineers routinely calculate a factor of safety (FS) to evaluate the stability of earth slopes. The Simplified Bishop method (Wright, et al, 1973) is a popular basis for computer analysis programs. A minimum FS of 1.3 is commonly considered as the design basis for most slopes. Failure is assumed to occur when the FS is less than 1.0.

Because the FS analysis does not have a way to consider the variability of the soil strength, the true safety of a slope is unknown. A reliability approach, where a probability of failure is calculated, is a better method for slope design because it accounts for variability in soil strengths.

The probability of slope failure method is based on the "Point Estimate Method" (PEM) which was developed by Rosenblueth (1975 and 1981) and described by Harr (1987). In the PEM method, a distribution of the variable must be found or assumed. If a normal distribution is assumed, the problem is simplified. Details of the PEM method and a discussion of other distributions are contained in a thesis by Garrett (1989) and a paper by McGuffy, Iori, Kyfor and Grivas (1981).

2 APPLICATION OF THE POINT ESTIMATE METHOD TO SLOPE STABILITY

2.1 MEAN AND STANDARD DEVIATION

To apply the PEM, the mean and standard deviation of the soil strength in each layer must be found. Soil strength may be cohesion, C, and/or internal friction, ϕ. Between layers, strength parameters are considered independent. Within a soil layer, however, the cohesion may be correlated to the internal friction.

$$Mean, \ \bar{x} = \frac{x_1 + x_2 + x_3 + \ldots x_n}{n} \qquad \text{(Eqn. 1)}$$

$$Standard \ Deviation, \ \sigma = \sqrt{\frac{\Sigma \ (x_i - \bar{x})^2}{(n - 1)}} \qquad \text{(Eqn. 2)}$$

where, x = the C or ϕ values in the layer
n = the number of C or ϕ (tests performed) values in the layer

2.2 CORRELATION COEFFICIENT

For each soil layer with C and ϕ, the correlation must be found. Correlations are dependent on the type of strength test. For the consolidated undrained triaxial test, Harr (1987) reports a correlation, r, of about +0.25. A positive correlation means the internal friction increases when the cohesion increases. The undrained triaxial test is the best predictor for quick failures caused by earthquakes or the sudden drawdown of water at a levee or dam. Drained triaxial tests have negative correlations and are usually the best predictor of field performance. Wolff reported a drained triaxial correlation of -0.47 (Harr, 1987). The correlation coefficient, r, is calculated by the following:

$$Correlation\ Coefficient,\ r = \frac{N\ \Sigma(C \cdot \phi) - \Sigma C\ \Sigma\phi}{\sqrt{[N\ \Sigma C^2 - (\Sigma C)^2\ ] \times [N\ \Sigma\phi^2 - (\Sigma\phi)^2]}} \qquad (Eqn.\ 3)$$

where, $\quad N$ = the number of strength tests

2.3 HIGH AND LOW STRENGTH VALUES

Variation in C and ϕ is accounted for by adding or subtracting the standard deviation. For example, a high cohesion, C+, is obtained by adding the standard deviation of the cohesion to the mean. A low cohesion, C-, is the mean less the standard deviation. In turn, $\phi+$ and $\phi-$ is the mean internal friction + or - the standard deviation of internal friction.

$$C+ = \overline{C} + \sigma_C \qquad (Eqn.\ 4a)$$
$$C- = \overline{C} - \sigma_C \qquad (Eqn.\ 4b)$$

where, $\quad \overline{C}$ = mean value of cohesion

$$\phi+ = \overline{\phi} + \sigma_\phi \qquad (Eqn.\ 5a)$$
$$\phi- = \overline{\phi} - \sigma_\phi \qquad (Eqn.\ 5b)$$

where, $\quad \phi$ = mean value of internal friction

2.4 SLOPE SAFETY FACTORS

Safety Factors must be found for all combinations of soil strength. The number of combinations is 2^n, where n is the number of variables (soil strengths). A slope with two layers, each layer with a C and ϕ, has 2^4 or 16 combinations of soil strength. The set of safety factors reflects the variation of soil strength. The symbol FS++++ is

used for a slope containing two soil layers with C+ and φ+ used for strength values in both layers. FS-++ is the symbol for the FS when C- and φ+ are used for the first layer and C+ and φ+ are used for the second layer.

2.5 WEIGHING FUNCTIONS

Weighing functions must be applied to the FS's. The weighing functions are point estimates, p, of the distribution of the FS's. The symbol p++++ is used for the point applied to FS++++ as described in section 2.4. The sum of the p's is equal to 1.

2.51 Independent Layers

For the case where each soil layer has only a C (a clay) or φ (a sand) the soil strengths are not correlated. If normal distribution is assumed, the point estimates are:

$$p = 1/2^n \hspace{3cm} \textit{(Eqn. 6)}$$

where,   *n = the number of variables (layers when each layer has only a C or φ).*

The points for two soil layers with C or φ are:

$$p++ = p+- = p-+ = p-- = 1/4$$

For three soil layers with C or φ, the points are:

$$p+++ = p++- = p+-+ = p-++ = p+-- = p-+- = p--+ = p--- = 1/8$$

The points for four soil layers are:

$$p++++ = p+++- = p++-+ = p+-++ = p-+++ = p++-- =$$
$$p+--+ = p--++ = p+--- = p-+-- = p--+- = p---+ = p-++- =$$
$$p+-+- = p---- = p-+-+ = 1/16$$

2.52 Correlated Layers

When a slope has a single layer with both C and φ (two variables), the points are:

$$p++ = p-- = (1 + r)/4 \hspace{3cm} \textit{(Eqn. 7a)}$$

$$p+- = p-+ = (1 - r)/4 \hspace{3cm} \textit{(Eqn. 7b)}$$

A slope that has two soil layers, each with C and ϕ (four variables), will have the following points:

$$p{+}{+}{+}{+} = p{-}{-}{-}{-} = p{+}{+}{-}{-} = p{-}{-}{+}{+} = (1+r_1+r_2)/16 \qquad \text{(Eqn. 8a)}$$

$$p{+}{+}{+}{-} = p{-}{-}{-}{+} = p{+}{+}{-}{+} = p{-}{-}{+}{-} = (1+r_1-r_2)/16 \qquad \text{(Eqn. 8b)}$$

$$p{+}{-}{-}{-} = p{-}{+}{+}{+} = p{+}{-}{+}{+} = p{-}{-}{+}{-} = (1-r_1+r_2)/16 \qquad \text{(Eqn. 8c)}$$

$$p{+}{-}{+}{-} = p{-}{+}{-}{+} = p{+}{-}{-}{+} = p{-}{+}{+}{-} = (1-r_1-r_2)/16 \qquad \text{(Eqn. 8d)}$$

2.53 Mixed Layers

For the case where there are two layers of soil, one layer contains either C or ϕ and the other contains both C and ϕ, the points are:

$$p{+}{+}{+} = p{+}{-}{-} = p{-}{-}{-} = p{-}{+}{+} = (1+r_2)/8 \qquad \text{(Eqn. 9a)}$$

$$p{+}{+}{-} = p{+}{-}{+} = p{-}{-}{+} = p{-}{+}{-} = (1-r_2)/8 \qquad \text{(Eqn. 9b)}$$

2.6 STANDARD DEVIATION OF THE FS'S

The expected value of the factor of safety, E[FS], and the expected value of the squared FS's, must be found in order to calculate the standard deviation of the FS, σ[FS].

2.61 Two Variables

For a slope with two variables (either two layers with C or ϕ, or one layer with C and ϕ):

$$E[FS] = p{+}{+}(FS{+}{+}) + p{+}{-}(FS{+}{-})$$
$$+ p{-}{+}(FS{-}{+}) + p{-}{-}(FS{-}{-}) \qquad \text{(Eqn. 10a)}$$

$$E[FS^2] = p{+}{+}(FS{+}{+})^2 + p{+}{-}(FS{+}{-})^2$$
$$+ p{-}{+}(FS{-}{+})^2 + p{-}{-}(FS{-}{-})^2 \qquad \text{(Eqn. 10b)}$$

$$\sigma[FS] = (E[FS^2] - E[FS]^2)^{.5} \qquad \text{(Eqn. 11)}$$

2.62 Three Variables

For a slope with three variables (either three layers with C or ϕ, or two layers; one layer with C or ϕ, and one layer with C and ϕ):

$$E[FS] = p{+}{+}{+}(FS{+}{+}{+}) + p{+}{+}{-}(FS{+}{+}{-})$$
$$+ p{+}{-}{-}(FS{+}{-}{-}) + p{-}{-}{-}(FS{-}{-}{-})$$
$$+ p{-}{-}{+}(FS{-}{-}{+}) + p{-}{+}{+}(FS{-}{-}{+})$$
$$+ p{-}{+}{-}(FS{-}{+}{-}) + p{+}{-}{+}(FS{+}{-}{+}) \qquad (Eqn.12a)$$

$$E[FS^2] = p{+}{+}{+}(FS{+}{+}{+})^2 + p{+}{+}{-}(FS{+}{+}{-})^2$$
$$+ p{+}{-}{-}(FS{+}{-}{-})^2 + p{-}{-}{-}(FS{-}{-}{-})^2$$
$$+ p{-}{-}{+}(FS{-}{-}{+})^2 + p{-}{+}{+}(FS{-}{+}{+})^2$$
$$+ p{-}{+}{-}(FS{-}{+}{-})^2 + p{+}{-}{+}(FS{+}{-}{+})^2 \qquad (Eqn.12b)$$

$$\sigma[FS] = (E[FS^2] - E[FS]^2)^{.5} \qquad (Eqn.11)$$

2.63 Four or More Variables

For four or more variables, the expected FS, E[FS], is found by multiplying the points, p, by their respective FS's and summing the products (see equations 10a and 12a).

The $E[FS^2]$ is found by multiplying the points, p, by their respective squared FS's and summing the products (see equations 10b and 12b).

The standard deviation is found from equation 11.

2.7 PROBABILITY OF FAILURE

For normal distribution, the standardized variable Z is:

$$Z = (FS - E[FS])/\sigma[FS] \qquad (Eqn. 13)$$

where, *FS = the cutoff value to be evaluated (FS = 1)*

E[FS] and σ[FS] are found from section 2.6.

With Z, the probability that the FS will be less than 1 can be found from the normal distribution table in Appendix A.

3 ACCEPTABLE FAILURE PROBABILITIES

In order to evaluate a design, the calculated probability of failure should be compared to an acceptable probability. A table of acceptable failure probabilities was proposed by Santamarina, et. al. (1992). A partial listing of the table is contained in Table 1.

TABLE 1. Slope Stability - Probability of Failure

CONDITIONS	P_f
Unacceptable in most cases	> 0.1
Temporary structures with low repair cost	0.1
Low consequences of failure repairs when time permits	0.02
Existing large cut on interstate highway	0.01
Acceptable in most cases except if lives may be lost	0.001
Acceptable for all slopes	0.0001
Unnecessarily low	0.00001

4 EXAMPLES

4.1 CONVERSION FACTORS

SI to English
1 m = 3.281 ft
1 kN/m² = 20.885 lb/ft²
1 kN/m³ = 6.361 lb/ft³

English to SI
1 ft = 0.3048 m
1 lb/ft² = 0.04788 kN/m²
1 lb/ft³ = 0.1572 kN/m³

4.2 TWO LAYERS WITH EITHER C OR Φ

Two examples using the slope in Figure 1 will show how the method is applied. The unit weight of both soil layers is 20 kN/m³
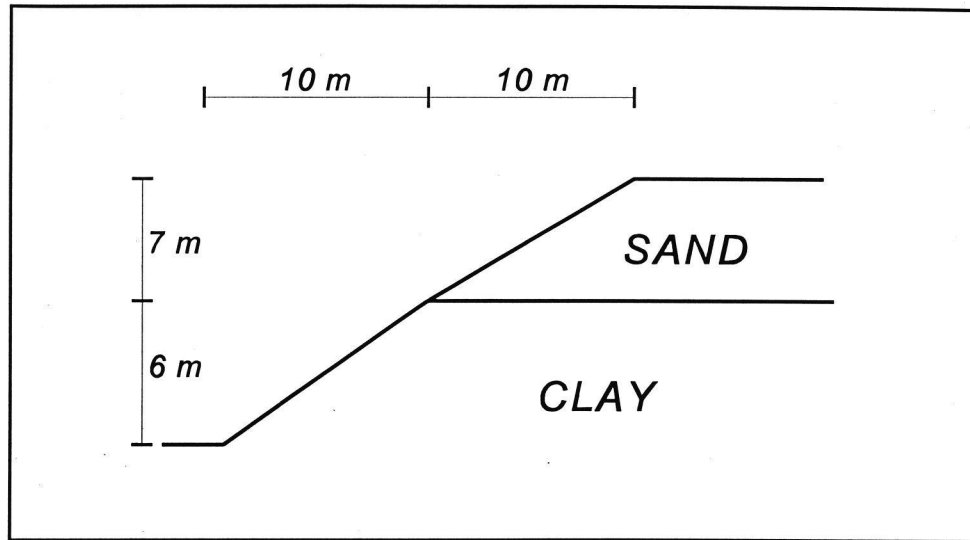
FIGURE 1

4.21 Example 1: Slope with Uniform Soils

The internal friction, φ, and cohesion, C, from tests for the soil in Figure 1 are:

SAND	CLAY
φ°	C (kN/m²)
33.5	60
36.5	63
35.5	64
34.5	58
35.1	62.5
34.9	

The **mean** (Eqn.1) and **standard deviation** (Eqn.2) are as follows:

SAND	CLAY
mean φ = 35°	mean C = 61.5 kN/m²
σ (φ) = 1°	σ (C) = 2.45

The **high and low strength values** (Eqn. 4a, 4b, 5a, and 5b) used to determine slope stability factors of safety are:

$$\phi+ = 36° \qquad C+ = 63.95 \text{ kN/m}^2$$
$$\phi- = 34° \qquad C- = 59.05 \text{ kN/m}^2$$

The **strength combinations** for slope stability analysis are:

$$
\begin{array}{lll}
++ \text{ Sand } \phi & = 36° & \text{Clay C} = 63.95 \text{ kN/m}^2 \\
+- \text{ Sand } \phi & = 36° & \text{Clay C} = 59.05 \text{ kN/m}^2 \\
-+ \text{ Sand } \phi & = 34° & \text{Clay C} = 63.95 \text{ kN/m}^2 \\
-- \text{ Sand } \phi & = 34° & \text{Clay C} = 59.05 \text{ kN/m}^2
\end{array}
$$

The resulting **factors of safety** from the computer program PCSTABL5 (Bishop Method) are:

$$
\begin{array}{ll}
++ \text{ FS} & = 1.350 \\
+- \text{ FS} & = 1.248 \\
-+ \text{ FS} & = 1.348 \\
-- \text{ FS} & = 1.246
\end{array}
$$

The **weighing functions** (Eqn. 6) for two soil types with 1 strength parameter per layer is:

$$ p++ = p+- = p-+ = p-- = 0.25 $$

The **expected FS** (Eqn. 10a) is:

$$
\begin{aligned}
E[FS] &= 0.25(1.350) + 0.25(1.248) + 0.25(1.348) + 0.25(1.246) \\
&= 1.298
\end{aligned}
$$

The **expected FS^2** (Eqn. 10b) is:

$$
\begin{aligned}
E[FS^2] &= 0.25(1.350)^2 + 0.25(1.248)^2 + 0.25(1.348)^2 + 0.25(1.246)^2 \\
&= 1.6874
\end{aligned}
$$

The **standard deviation of the FS's** (Eqn. 11) is:

$$
\begin{aligned}
\sigma[FS] &= [(1.687) - (1.298)^2)]^{.5} \\
&= 0.051
\end{aligned}
$$

The **standardized variable** (Eqn. 13) is:

$$ Z = (1-1.298)/0.051 = -5.84 $$

For a FS = 1, where failure is assumed to occur, the **probability of failure, P_f**, is (Appendix A):

$$ P_f < 0.0000001 $$

This probability of failure, according to Table 1, is unnecessarily low.

4.22 Example 2: Slope with Variable Clay

Strength test results for the soil in Figure 1 are as follows:

Sand $\phi°$	Clay $C(kN/m^2)$
36.5	55
34	50
34.5	71
35.5	82
34.5	53
	58

The **mean** (Eqn. 1) and **standard deviation** (Eqn. 2) are as follows:

Sand	Clay
mean $\phi = 35°$	mean C = 61.5 kN/m^2
$\sigma (\phi) = 1°$	$\sigma (C) = 12.4$ kN/m^2

The **high and low strength values** (Eqn. 4a, 4b, 5a, and 5b) used to determine slope stability factors of safety are:

$$\phi+ = 36° \qquad C+ = 73.9 \text{ kN/m}^2$$
$$\phi- = 34° \qquad C- = 49.1 \text{ kN/m}^2$$

The **strength combinations** for slope stability analysis are:

++	Sand $\phi = 30°$	Clay C = 73.9 kN/m^2
+-	Sand $\phi = 36°$	Clay C = 49.1 kN/m^2
-+	Sand $\phi = 34°$	Clay C = 73.9 kN/m^2
--	Sand $\phi = 34°$	Clay C = 49.1 kN/m^2

The **factors of safety** from the computer program PCSTABL5 (Bishop Method) are:

$$++ \text{ FS } = 1.556$$
$$+- \text{ FS } = 1.040$$
$$-+ \text{ FS } = 1.554$$
$$-- \text{ FS } = 1.039$$

The **expected FS** (Eqn. 10a) is:

$$E[FS] = 1.297$$

The **standard deviation of the FS's** (Eqn. 11) is:

$$\sigma[FS] = 0.2578$$

The **standardized variable** (Eqn. 13) and the **probability of failure** (Appendix A) are:

$$Z = (1-1.297)/0.2578 = -1.1637$$
$$P_f = 0.125$$

This probability of failure, according to Table 1 is too high, even for temporary structures with low repair costs.

4.23 Example Comparison

The probability that the slopes in the two examples would fail is greatly different; less than 0.00001% for the first example vs. 12.5% for the second example. This difference is surprising because the geometry unit weight, and average strength of the soil layers within the slopes are the same. The reason for the difference in probability of failure is the variability in cohesion of the clay layer. In the uniform clay layer (section 4.11) the standard deviation of the cohesion is 2.45 kN/m^2 or 4% of the average cohesion. The variable clay layer (section 4.12) has a standard deviation of 12.4 kN/m^2 or 20% of the average cohesion.

4.3 EXAMPLE 3: THREE LAYERS WITH EITHER C OR Φ

The figure below is a slope on Interstate 40 near Morrilton, Arkansas. The slope has been divided into 3 layers.
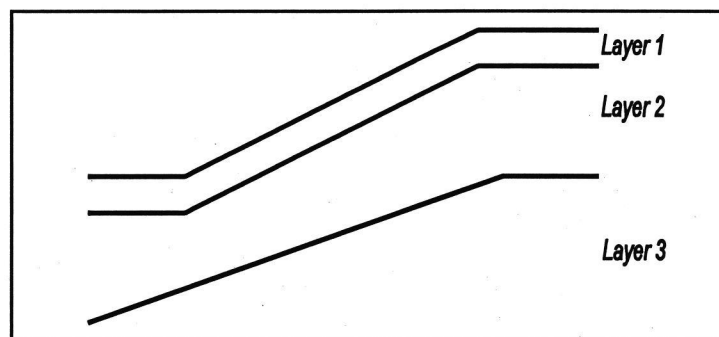

FIGURE 2

In this example, the only strength parameter in each layer is cohesion. From the strength tests, the **mean** and **standard deviation** of each layer obtained from Eqn. 1 and 2 are:

LAYER NO.	MEAN STRENGTH	STANDARD DEVIATION
1	180 lb/ft^2	16 lb/ft^2
2	410 lb/ft^2	54 lb/ft^2
3	600 lb/ft^2	138 lb/ft^2

The **high and low values** (Eqn. 4a and 4b) for cohesion in lb/ft^2 are:

Layer 1	Layer 2	Layer 3
$C_1+ = 196$	$C_2+ = 464$	$C_3+ = 738$
$C_1- = 164$	$C_2- = 356$	$C_3- = 462$

The next step is putting together the strength combinations. In this case, since there are 3 strength parameters, the are 2^3, or 8 strength combinations.

The **strength combinations** and **factors of safety** from the computer program PCSTABL5 (Bishop Method) for each combination are as follows:

COMBINATION	C_1	C_2	C_3	FS
+++	196	464	738	1.466
++-	196	464	462	1.293
+-+	196	356	738	1.145
-++	164	464	738	1.452
+--	196	356	462	1.131
-+-	164	464	462	1.285
--+	164	356	738	1.145
---	164	356	462	1.131

The next step is the calculation of the **expected FS** (Eqn. 12a), **expected value of the squared FS's** (Eqn. 12b), and **standard deviation of the FS's** (Eqn. 11):

$$E[FS] = 1.256$$
$$E[FS^2] = 1.595$$
$$\sigma[FS] = 0.1326$$

Then the **standardized variable** (Eqn. 13) is found for a FS = 1.

$$Z = (1-1.256)/0.1326 = 1.93$$

By using this Z and the probability chart in Appendix A, the **probability of failure** for this slope is 2.68%.

4.4 EXAMPLE 4: FOUR LAYERS WITH C OR Φ

The example for four layers of soil is taken from the thesis at the University of Arkansas by Steven Garrett (1989). Figure 3 contains the geometry of the slope.
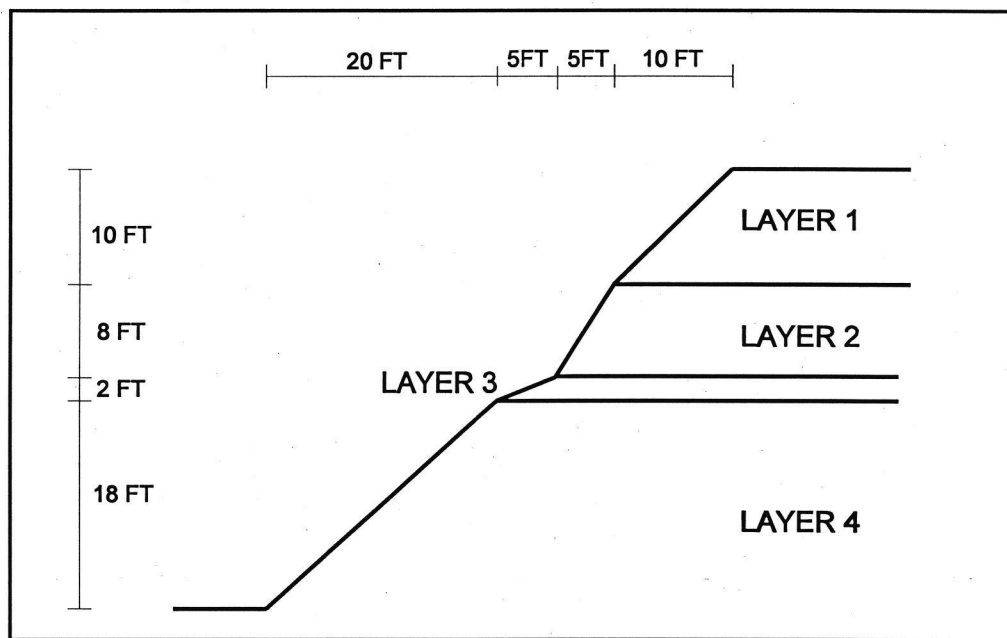

FIGURE 3

In this example, the first and third layers are clay and the second and fourth layers are sand. From the strength tests, the **mean** (Eqn.1) and **standard deviation** (Eqn. 2) of strengths are:

LAYER	MEAN STRENGTH	STANDARD DEVIATION
1	3500 lb/ft^2	200 lb/ft^2
2	27°	5°
3	2000 lb/ft^2	300 lb/ft^2
4	32°	2.5°

The **high and low strength values** (Eqn. 4a, 4b, 5a, and 5b) are:

LAYER	HIGH STRENGTH	LOW STRENGTH
1	3700 lb/ft^2	3300 lb/ft^2
2	32°	22°
3	2300 lb/ft^2	1700 lb/ft^2
4	34.5°	29.5°

The **strength combinations** and **factors of safety** from the computer program PCSTABL5 (Bishop Method) for each combination are:

COMBINATION	C_1 lb/ft^2	ϕ_2 °	C_3 lb/ft^2	ϕ_4 °	FS
+ + + +	3700	32	2300	34.5	1.4024
+ + + -	3700	32	2300	29.5	1.1966
+ + - -	3700	32	1700	29.5	1.1428
+ - - -	3700	22	1700	29.5	1.1239
- - - -	3300	22	1700	29.5	1.1235
- + - -	3300	32	1700	29.5	1.1424
- + + -	3300	32	2300	29.5	1.1966
- + + +	3300	32	2300	34.5	1.4021
- - + -	3300	22	2300	29.5	1.1798
- - + +	3300	22	2300	34.5	1.3786
- - - +	3300	22	1700	34.5	1.1798
- + - +	3300	32	1700	34.5	1.3352
+ - + -	3700	22	2300	29.5	1.1798
+ - - +	3700	22	1700	34.5	1.3130
+ - + +	3700	22	2300	34.5	1.3790
+ + - +	3700	32	1700	34.5	1.3356

The **expected FS, expected value of the squared FS's,** and **standard deviation of the FS's** are found per article 2.63:

$$E[FS] = 1/16 \, [1.4024 + 1.1966 + 1.1428 + 1.1239 + 1.1235 + 1.1424 +$$
$$1.1966 + 1.4021 + 1.1798 + 1.3786 + 1.3126 + 1.3352 +$$
$$1.1798 + 1.3130 + 1.3790 + 1.3356]$$
$$= 1.2590$$

$$E[FS^2] = 1/16 \, [1.4024^2 + 1.1966^2 + 1.1428^2 + 1.1239^2 + 1.1235_2 + 1.1424^2 +$$
$$1.1966^2 + 1.4021^2 + 1.1798^2 + 1.3786^2 + 1.3126^2 + 1.3352^2 +$$
$$1.1797^2 + 1.3130^2 + 1.3790^2 + 1.3356^2]$$
$$= 1.5958$$

$$\sigma[FS] = (1.5958 - 1.2590^2)^{.5}$$
$$= 0.1035$$

Then the **standardized variable** is found for a FS = 1.

$$Z = (1.0 - 1.2590)/0.1305$$
$$= 2.50$$

Using the probability chart in Appendix A, the **probability of failure** is 0.62%.

4.5 EXAMPLE 5: ONE SOIL WITH TWO VARIABLES

This example is taken from a paper by Verduin and Lovell (1988). The embankment is 40 feet high and is built on a slope of two horizontal to one vertical (Figure 4). The soil has a unit weight of 140 lb/ft^3.
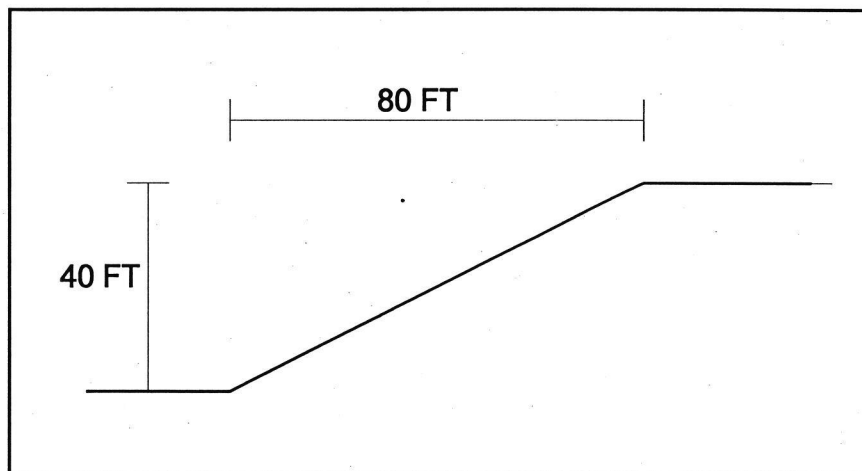

FIGURE 4

The **mean** and **standard deviation** of the soil strength are:

$$\text{mean } C = 200 \text{ lb/ft}^2 \qquad \sigma(C) = 80 \text{ lb/ft}^2$$
$$\text{mean } \phi = 25° \qquad \sigma(\phi) = 2.5°$$

The **correlation coefficient** (Eqn. 3) as determined from laboratory tests is +0.25.

The **high and low strength values** (Eqn 4a, 4b, 5a, and 5b) used to determine slope stability factors of safety are:

$$\phi+ = 25 + 2.5 = 27.5° \qquad C+ = 200 + 80 = 280 \text{ lb/ft}^2$$
$$\phi- = 25 - 2.5 = 22.5° \qquad C- = 200 - 80 = 120 \text{ lb/ft}^2$$

The slope **factors of safety** from the computer program PCSTABL5 (Bishop Method) are:

$$FS++ = 1.685$$
$$FS+- = 1.454$$
$$FS-+ = 1.373$$
$$FS-- = 1.140$$

The **weighing functions** (Eqn. 7a and 7b) are:

$$p++ = p-- = 0.25(1+0.25) = 0.3125$$
$$p+- = p-+ = 0.25(1-0.25) = 0.1875$$

The **expected FS** (Eqn. 10a), **expected value of the squared FS's** (Eqn.10b), and **standard deviation of the FS's** (Eqn. 11) are:

$$\begin{aligned} E[FS] &= 0.3125(1.685) + 0.1875(1.454) + \\ &\quad 0.1875(1.373) + 0.3125(1.140) \\ &= 1.413 \end{aligned}$$

$$\begin{aligned} E[FS^2] &= 0.3125(1.685)^2 + 0.1875(1.454)^2 + \\ &\quad 0.1875(1.373)^2 + 0.3125(1.140)^2 \\ &= 2.043 \end{aligned}$$

$$\begin{aligned} \sigma[FS] &= (2.043 - (1.413)^2)^{.5} \\ &= 0.216 \end{aligned}$$

The **standardized variable** (Eqn. 13) and **probability of failure** (Appendix A) are:

$$Z = (1.0 - 1.413)/0.216 = -1.91$$
$$P_f = 2.8\%$$

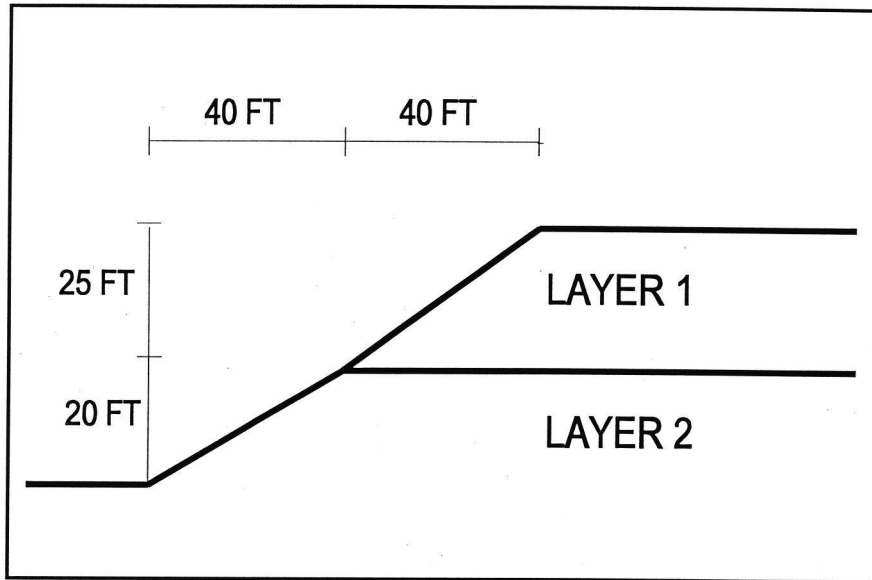## 4.6 TWO SOIL LAYERS WITH TWO VARIABLES EACH

FIGURE 5

Unit Weight Layer 1 = 110 lb/ft^3
Unit Weight Layer 2 = 120 lb/ft^3

The **mean** (Eqn. 1) and **standard deviation** (Eqn. 2) of the soil strength are:

First Layer		Second Layer	
C (lb/ft^2)	$\phi°$	C (lb/ft^2)	$\phi°$
200	31	150	27
180	33	110	30
210	28	240	24
230	27	220	25
160	34	120	32

Layer 1   mean C = 196 lb/ft^2 $\sigma(C)$ = 27 lb/ft^2
           mean ϕ = 30.6° $\sigma(\phi)$ = 3.05°

Layer 2   mean C = 168 lb/ft^2 $\sigma(C)$ = 58.9 lb/ft^2
           mean ϕ = 27.6° $\sigma(\phi)$ = 3.36°

The **correlation coefficients** (Eqn. 3) are -0.964 for layer 1 and -0.927 for layer 2.

The **high and low strength values** (Eqn. 4a, 4b, 5a, and 5b) are:

$$C_1+ = 223 \text{ lb/ft}^2 \qquad \phi_1+ = 33.65°$$
$$C_1- = 169 \text{ lb/ft}^2 \qquad \phi_1- = 27.55°$$
$$C_2+ = 226.9 \text{ lb/ft}^2 \qquad \phi_2+ = 30.96°$$
$$C_2- = 109.1 \text{ lb/ft}^2 \qquad \phi_2- = 24.24°$$

The slope **factors of safety** from the computer program PCSTABL5 (Bishop Method) are:

$$FS++++ = 1.6235$$
$$FS+++- = 1.3798$$
$$FS++-- = 1.2123$$
$$FS+--- = 1.1714$$
$$FS---- = 1.1413$$
$$FS---+ = 1.3573$$
$$FS--++ = 1.5226$$
$$FS-+++ = 1.5897$$
$$FS-+-+ = 1.4579$$
$$FS+-+- = 1.3295$$
$$FS+--+ = 1.3977$$
$$FS-++- = 1.3527$$
$$FS++-+ = 1.4560$$
$$FS--+- = 1.3014$$
$$FS+-++ = 1.5595$$
$$FS-+-- = 1.1873$$

The **weighing functions** (Eqn. 8a, 8b, 8c, and 8d) are:

$$p++++ = p---- = p++-- = p--++ = (1-0.964-0.927)/16 = -0.05569$$
$$p+++- = p---+ = p++-+ = p--+- = (1-0.964+0.927)/16 = 0.0602$$
$$p+--- = p-+++ = p+-++ = p-+-- = (1+0.964-0.927)/16 = 0.0648$$
$$p+-+- = p-+-+ = p+--+ = p-++- = (1+0.964+0.927)/16 = 0.1807$$

The **expected FS, expected value of the squared FS's,** and **standard deviation of the FS's** are:

$$
\begin{aligned}
E[FS] = {} & -0.05569(1.6235) + 0.0602(1.333798) + -0.05569(1.2123) \\
& + 0.0648(1.1714) + -0.05569(1.1413) + 0.0602(1.3573) \\
& + -0.05569(1.5226) + 0.0648(1.5897) + 0.1807(1.4579) \\
& + 0.1807(1.3527) + 0.1807(1.3977) + 0.0602(1.4560) \\
& + 0.0602(1.3014) + 0.1807(1.3295) + 0.0648(1.5595) \\
& + 0.0648(1.1873) \\
= {} & 1.3821
\end{aligned}
$$

$$E[FS^2] = -0.05569(1.6235_2) + 0.0602(1.3798_2) + -0.05569(1.2123_2)$$
$$+ 0.0648(1.1714_2) + -0.05569(1.1413_2) + 0.0602(1.3573_2)$$
$$+ -0.05569(1.5226_2) + 0.0648(1.5897_2) + 0.1807(1.4578_2)$$
$$+ 0.1807(1.3527_2) + 0.1807(1.3977_2) + 0.0602(1.4560_2)$$
$$+ 0.0602(1.3014_2) + 0.1807(1.3295_2) + 0.0648(1.5595_2)$$
$$+ 0.0648(1.1873_2)$$
$$= 1.9136$$

$$\sigma[FS] = (1.9136 - 1.3821^2)^{.5} = 0.05798$$

The **standardized variable** (Eqn. 13) is:

$$Z = (1.0 - 1.382)/0.058$$
$$= -6.59$$

The **probability of failure** (Appendix A) is less than .003%.

5 CONCLUSION

The reliability approach to slope stability is superior to the safety factor approach because it accounts for variability in soil strength.
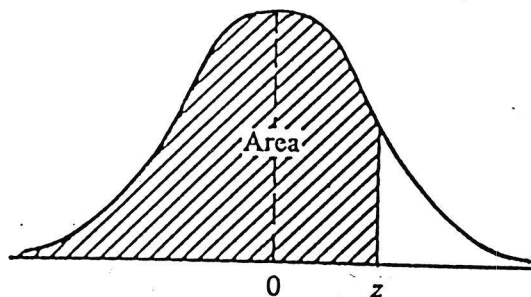

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APPENDIX A

NORMAL DISTRIBUTION CURVE AREAS

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
−3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
−3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
−3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
−3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
−3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
−2.9	0.0019	0.0018	0.0017	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
−2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
−2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
−2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
−2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
−2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
−2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
−2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
−2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
−2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
−1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
−1.8	0.0359	0.0352	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
−1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
−1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
−1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
−1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0722	0.0708	0.0694	0.0681
−1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
−1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
−1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
−1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
−0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
−0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
−0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
−0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
−0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
−0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
−0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
−0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
−0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
−0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641